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Bayesian Analysis of Threshold Autoregressive Model with First Order Autoregressive Innovations

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ABSTRACT

Financial assets exhibit dramatic changes in behaviour. This work examined a two-regime Threshold autoregressive (TAR) models when the innovations follow a first-autoregressive order process. The Bayesian method is proposed to build in the linear first-order autoregressive process with identical distributed innovations. The practical usefulness of this method is demonstrated with simulated and real-life data using U.S.A quarterly real GDP as an example. In simulation experiments and real life example, an increase in first order process parameter, ρ value leads to better estimates in the proposed model. Also, the proposed model was compared with TAR model where the disturbance term does not exhibit regime switching. The proposed model performed well than the traditional TAR model using the simulated and real life data. An increase in first order process parameter, ρ will lead to better estimates and forecast. Hence, the proposed model performed well.

Keywords: Simulation, regime, GDP, simulation, real life data, Innovations.

1 Introduction

There are many macroeconomics variables that may vary over business cycle. Threshold autoregressive (TAR) models belong to such class of models that has different autoregressive representations in different regimes. Due to importance of TAR models to practitioners, numerous studies have been conducted, and several statistical inferences about the model have also been established. For instance, a linear approximation of the nonlinear TAR model was proposed by Giordano et al. (2022), called Linear- AIC to estimate the autoregressive order of the two regimes while Djeddour and Bou (2003) applied TAR model to forecast U.S. export from January, 1991 to December, 2004. They considered thresholds models that are special cases of procedure for non-linear models on average above TAR.

Different works on TAR model were also discussed in the context of Bayesian inference. Xia et al. 2012 considered Bayesian analysis of threshold autoregressive moving average model with exogenous inputs (TARMAX). Two different MCMC methods (Gibbs sampler and metropolis algorithms) were used. One method was used to obtain iterative least squares estimates while the second method was used to estimate desired marginal posterior distributions. Safadi and Morettin (2000) also applied a Bayesian analysis to TAR moving average models in the case of two regimes while Ojo (2021) analyzed Nigerian inflation rate with Bayesian TAR model with special case where a variable trigger the first lag of dependent variable.

Bayesian analysis was carried out on a multivariate TAR model with missing data by Colderon et al. (2017) while a procedure for forecasting with multivariate TAR models was proposed by Calderon and Nieto (2021) through the predictive distribution using Bayesian approach. The strategy gave the forecasts for response and exogenous variables while the coverage percentages of the forecast intervals and variability of the predictive distributions were also considered.

Since the invention of TAR model by Tong (1983), different TAR models that exhibit different regime switching have been in existent. More importantly, attention has been drawn by researchers in knowing whether the error term of TAR model exhibits regime switching behaviour (see Chen (1988), Pan et al. (2017)). This invention started with the work of Chen (1988). Chen (1988) constructed a generalized framework for TAR model that made TAR model to be flexible in applications. In order to implement the proposed method, two series of quarterly macroeconomic variables (GNP and M1) were used.

A method of Bayesian stochastic search selection was introduced by Pan et al. (2017) to identify a threshold-dependent sequence with highest probability in a threshold autoregressive model. The innovation of the method was introduced to estimate the TAR model without assuming the fixed number of threshold values, thereby making it more flexible and useful. A hybrid Markov chain Monte Carlo method, which combines Metropolis-Hastings algorithm and Gibbs sampler were used to compute model parameters. Both simulation experiments and real data example demonstrated the usefulness of the proposed approach.

Bayesian analysis was carried out on multiple regimes of threshold autoregressive model with possible break points by Agiwal and Kumar (2020). Both Gibbs sampler and Metropolis–Hastings algorithm were applied to compute the estimates of parameter while both real data and simulated was tested for empirical evidence. Prediction of future values with the use of autoregressive process in disturbance term is of great importance to time series econometricians and practitioners. Apart from predicting the future values of series, autoregressive models helps under certain market conditions, such as when we have financial crises or periods of rapid technological changes.

In this paper, we offer a Bayesian inference and estimator technique on threshold models that switches on the error term with autoregressive model. We will also highlight the strength and weaknesses of the procedure. The remainder of the paper is structured as follows. Section 2 gives the methodology which comprises the threshold model that incorporates the first order autoregresive, Bayesian method of estimation and numerical examples that involve both the Monte Carlo simulation and real life situations. Section 3 presents and discusses the results while Section 4 concludes.

2 Methodology

2.1 The Threshold autoregressive model

Consider a two-regime TAR model with a time series variable given as:

$$y_t = \alpha_{10} + \alpha_{11} y_{t-1} + ... + \alpha_{1m} y_{t-m} + u_t. \quad if Z_{t-d} \leq \lambda \tag{1}$$

$$y_t = \alpha_{20} + \alpha_{21} y_{t-1} + ... + \alpha_{2m} y_{t-m} + u_t. \quad if Z_{t-d} \leq \lambda \tag{2}$$

where $u_t \sim N(0, h^{-1})$ and $\alpha_j = (\alpha_{j0}, \alpha_{j1}, ..., \alpha_{jm})'$
Also, d is the delay parameter, Z_{t-d} is functions of the lags. The order of autoregressive for (1) and (2) is P=4.
Thus, threshold trigger can be simply be defined as:

$$Z_{t-d} = \frac{\sum_{d=1}^{P} y_{t-d}}{d} \tag{3}$$

If the error term follows a first autoregression, equation (1) and (2) become:

$$y_t = \alpha_{10} + \alpha_{11} y_{t-1} + ... + \alpha_{1m} y_{t-m} + \rho u_{t-1} + \epsilon. \quad if Z_{t-d} \leq \lambda \tag{4}$$

$$y_t = \alpha_{20} + \alpha_{21} y_{t-1} + ... + \alpha_{2m} y_{t-m} + \rho u_{t-1} + \epsilon. \quad if Z_{t-d} \leq \lambda \tag{5}$$

Assume d is an unknown parameter where $\epsilon_t \sim N(0, h^{-1}\Omega)$, and

$$\Omega = \frac{1}{1-\rho^2} \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

The model in (4) and (5) is flexible to accommodate some interesting practical importance as follows:

1. It will predict future security prices.

2. It will become a non-homogenous regression model, if the variance is different for each of the regimes.

3. It will be in form of regime model.

The main reason for transforming the model by including of first autoregressive series is because there is interest in the volatility of macroeconomic variables in recent time(s) and, in particular, the disturbance term does exhibits regime-switching behaviour.

2.2 Bayesian Inference

In this section, Bayesian estimation procedure will be carried out. We will derive the Posterior distribution when the TAR model has switches in disturbance term with first order autoregression while derivation for model in (1) and (2) have been shown in the works of Koop and Potter (2003), Koop et al. (2005), and Ojo (2021).
The regimes in (4) and (5) can simply be written as:

$$y_j = x_j\alpha_j + \epsilon_j \tag{6}$$

where y_j denotes the data in the jth regime for j = 1, 2. The matrix x_j contains an intercept and lags of the dependent variable for observations in the jth regime, and ϵ_j are errors, for j =1,2. The derivation of posterior distribution for model in (6) will follow the same manner of posterior distribution for normal linear regression model obtained by Adepoju and Ojo (2108).

The likelihood function of (6) is given as:

$$L(\alpha_j, h) = \frac{h_j^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \exp(-\frac{h}{2}(y_j - x_j\alpha_j)'(y_j - x_j\alpha_j)) \tag{7}$$

Here, a suitable prior would be considered for Bayesian inference. However, is worthwhile to briefly discuss the prior elicitation in TAR model, since this has received great attention in literature. Prior elicitation transforms domain knowledge of various kinds into well-defined prior distributions Mikkola, et al. 2023. In this work, we will focus on a familiar prior known as conjugate prior by translating existing knowledge into action.

The familiar natural conjugate prior, Normal-Gamma prior (Koop and Potter 2003, Koop et al. 2005) and non-informative prior will be extended. Assuming that parameter d is unknown with non-informative prior over 1, . . . ,m, that is, P(m=i)=$\frac{1}{m}$. The non-informative of this will be achieved by setting the hyperparamters to zero.

Also, assume a Normal Gamma prior is used for each of the regime we have;

$$P(\alpha_j, h_j) \sim NG(\alpha_j^o, Q_j^o, \frac{1}{S_j^o 2}, v_j^o) \quad j = 1, 2. \tag{8}$$

Using Bayes theorem, the density of the conjugate prior will be:

$$\begin{aligned} P(\alpha_j, h_j) &= P(\alpha_j|h_j)P(h_j) \\ &= \frac{h^{\frac{v_j^o+k}{2}-1}}{2\pi^{\frac{k}{2}}|Q_j^o|^{\frac{1}{2}}\Gamma\left(\frac{v_j^o}{2}\right)\left(\frac{2S_j^o}{v_j^o}\right)^{\frac{v_j^o}{2}}} \\ &\times \exp\left(-\frac{h}{2}(\alpha_j - \alpha_j^o)'(Q_j^o)^{-1}(\alpha_j - \alpha_j^o) + \frac{v_j^o}{S_j^o - 2}\right) \end{aligned} \tag{9}$$

where

$$P(\alpha_j|h_j) = \frac{h^{\frac{k}{2}}}{2\pi^{\frac{k}{2}}|Q_j^o|^{\frac{1}{2}}}[\exp(-\tfrac{h}{2}(\alpha_j - \alpha_j^o)'(Q_j^o)^{-1}(\alpha_j - \alpha_j^o)]$$

and

$$P(h_j) = \frac{1}{\Gamma(\frac{v_j^o}{2})(\frac{(2S_j^o 2}{v_j^o})^{\frac{v_j^o}{2}}} h^{\frac{v_j^o}{2}-1}[\exp(\frac{-hv_j^o}{2S_j^o - 2})]$$

Equation (9) is simply the distribution of prior known as multivariate Normal-Gamma prior. Posterior distribution summarises one's updated knowledge in Bayesian inference by balancing prior knowledge with observed data. Since the prior is a natural conjugate prior, posterior distribution will also have the same functional form as the prior. The prior used has the advantage that analytical results are available so that posterior simulator is not required. Posterior distribution can be obtained from the relation given below:

$$P(\alpha_j, h_j|\lambda, m_j) \propto L(\alpha_j, h_j)P(\alpha_j, h_j) \tag{10}$$

Combining equation (7) with (8), we have the posterior distribution given as:

$$= \frac{h_j^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \exp\left(-\frac{h}{2}(y_j - x_j\alpha_j)'(y_j - x_j\alpha_j)\right)$$
$$\times \frac{h^{\frac{v_j^o+k}{2}-1}}{2\pi^{\frac{k}{2}}|Q_j^o|^{\frac{1}{2}}\Gamma\left(\frac{v_j^o}{2}\right)\left(\frac{2S_j^o}{v_j^o}\right)^{\frac{v_j^o}{2}}} \tag{11}$$
$$\times \exp\left(-\frac{h}{2}(\alpha_j - \alpha_j^o)'(Q_j^o)^{-1}(\alpha_j - \alpha_j^o) + \frac{v_j^o}{S_j^o - 2}\right)$$

Ignoring the terms that do not depend on α_j and h_j, we have the same functional form as the prior distribution.
Hence, the posterior distribution is:

$$P(\alpha_j, h_j|\lambda, m_j) \sim NG(\alpha_j^*, Q_j^*, \frac{1}{S_j^*2}, v_j^*) \quad j = 1, 2. \tag{12}$$

where

$$Q_j^* = \frac{1}{(\frac{1}{\alpha_j}x_j'x_j)}$$
$$\alpha_j^* = Q_j^*(\frac{1}{Q_j^o}\alpha_j^o + x_j'x_j\hat{\alpha}_j)$$
$$v_j^* = T_j + v_j^o \text{ (T_j is member of observation)}$$
$$S_j^*2 = \frac{v_j^o S_j^o 2 + SSE + (\hat{\alpha}_j - \alpha_j^o)x_j'x_j Q_j^* \frac{1}{Q_j^o}(\hat{\alpha}_j - \alpha_j^o))}{v_j^o}$$

SSE is the sum of squared error and $\hat{\alpha}_j$ is the ordinary least squares estimator (see Adepoju and Ojo (2018) for more details)
The hyper parameters are set to be:
$\alpha_j = 0, k = 0.05, (S_j^o)^2 = 1, v_j^o = 2, Q_j^o = kI_o$, where I_o is an identity matrix. First autoregressive parameter, $0 < \rho < 1$; ρ are set to be: 0.2, 0.5, and 0.9.

2.3 Numerical analysis

To implement the proposed model, the empirical evidence using both simulated and real-life data will be illustrated for the performance of the proposed model. We will compare the properties of our Bayesian model estimation procedures for the proposed model with TAR model where the disturbance term does not exhibit regime switching.

2.3.1 Simulation study

We consider a data generating procedure used by Chen and Lee (1995) in this simulation study. It follows as:

$$y_t = 0.45 + 0.05y_{t-1} + 0.2y_{t-2} + 0.18y_{t-3} + 0.15y_{t-4} + \rho u_{t-1} + \epsilon. \quad \text{if } Z_{t-d} \leq \lambda \tag{13}$$

$$y_t = 0.8 + 0.65y_{t-1} - 0.5y_{t-2} - 0.3y_{t-3} + 0.09y_{t-4} + \rho u_{t-1} + \epsilon. \quad \text{if } Z_{t-d} \leq \lambda \tag{14}$$

In this simulation we set the number of observations to be n= 200. The time series data were generated from multivariate Gaussian distribution with means and specified variance-covariance. The coefficient vector was set to be in equation (8) while the innovations, were also simulated from Gaussian distribution. This will enable the simulated data set to be used in the proposed model.

2.3.2 Real data analysis

To illustrate the usefulness of this proposed model, we provide a real-life data example to illustrate the proposed model in application. We consider quarterly real GDP data of U.S.A. The data contains 244 values of real GDP between first quarter of year 1960 and last quarter of year 2021. We use the same prior settings as used in the simulation section.

3 Results and Discussion

In this Section, results of both simulated and real-life data will be presented when there is regime switching in error term of TAR model by first autoregressive process. Table 1 shows the simulation results obtained by using the process illustrated in Sub-section 4.1. The mean and standard deviation for the first and second regimes were recorded for all the parameters of the model given in (8). All the mean estimates are close to the true parameter values for the first and second regimes across all the rho values considered. However, when $\rho = 0.9$, the mean estimates for the regimes (first and second) are closer to the true parameter values than the other P values.

Also, the standard deviation for all parameters of the first regimes when $\rho = 0.9$ are smaller than when $\rho = 0.5$ and $\rho = 0.2$, except for the intercept and parameters. On the contrary, as the p decreases, Standard deviations for all parameters for the second regimes decreases as p decreases.

Table 2 presents posterior results for parameter estimates for each regime. As the ρ increases, the mean of the first and second regimes also increases for all the parameters of the model. However, the standard deviation decreases as the ρ value increases for all the regimes of the model. In Tables 3 and 4, posterior mean and standard deviation of TAR model without the first autoregressive process in the disturbance term are presented for simulated and real life, respectively. Comparing with Tables 3 and 4 (TAR model without the first autoregressive process) with Tables 1 and 2 (TAR model with the first autoregressive process) shows that results of TAR model when the disturbance term follows a first autoregressive process are better than TAR model that the disturbance term does not follows a first autoregressive process. Figures 1 and 2 show the plots of posterior for delay parameter, d. Since different values of d imply different threshold triggers, which makes the interpretation of the threshold to differs across d. Hence, the posterior for d allocates most of the probability to d = 1, and d = 3, for simulated and real life data, respectively.

Table 1: Posterior results using simulated data for TAR model with switches in error term having first autoregressive.

Rho	FRP[a]	Mean	SD	SRP[b]	Mean	SD
$\rho = 0.2$	α_{10}= 0.45	0.5360	0.2861	α_{20}= 0.8	0.2581	0.2993
$\rho = 0.5$	α_{10}= 0.45	0.5822	0.2737	α_{20}= 0.8	0.6138	0.4338
$\rho = 0.9$	α_{10}= 0.45	0.4230	0.4015	α_{20}= 0.8	0.7899	0.5197
$\rho = 0.2$	α_{11}= -0.05	-0.2300	0.2542	α_{21} = -0.65	-0.8745	0.1481
$\rho = 0.5$	α_{11}= -0.05	-0.1349	0.2798	α_{21} = -0.65	-0.7855	0.1566
$\rho = 0.9$	α_{11}= -0.05	-0.0554	0.2596	α_{21} = -0.65	-0.6675	0.1640
$\rho = 0.2$	α_{12}=0.2	0.0119	0.2865	α_{22}=-0.5	-0.7460	0.1747
$\rho = 0.5$	α_{12}=0.2	0.1262	0.3073	α_{22}=-0.5	-0.6514	0.1837
$\rho = 0.9$	α_{12}=0.2	0.1978	0.2656	α_{22}=-0.5	-0.5227	0.1862
$\rho = 0.2$	α_{13}=0.18	0.0107	0.2649	α_{23}=-0.3	-0.4763	0.1744
$\rho = 0.5$	α_{13}=0.18	0.1112	0.2679	α_{23}=-0.3	-0.4040	0.1890
$\rho = 0.9$	α_{13}=0.18	0.1786	0.2285	α_{23}=-0.3	-0.2841	0.2091
$\rho = 0.2$	α_{14}=0.15	0.0052	0.2482	α_{24}=0.09	-0.1008	0.2072
$\rho = 0.5$	α_{14}=0.15	0.1002	0.2679	α_{24}=0.09	-0.0325	0.2402
$\rho = 0.9$	α_{14}=0.15	0.1807	0.1976	α_{24}=0.09	0.0819	0.2659

[a]FRP - First Regime Parameter

[b]SRP - Second Regime Parameter

Table 2: Posterior results using real life data for TAR model with switches in error term having first autoregressive.

Rho	FRP[a]	Mean	SD	SRP[b]	Mean	SD
$\rho = 0.2$	α_{10}	0.3921	0.1675	α_{20}	-0.0881	0.1396
$\rho = 0.5$	α_{10}	0.4389	0.2090	α_{20}	-0.0434	0.2021
$\rho = 0.9$	α_{10}	0.3405	0.2877	α_{20}	0.0263	0.2388
$\rho = 0.2$	α_{11}	-0.2364	0.2132	α_{21}	-0.2701	0.1458
$\rho = 0.5$	α_{11}	-0.1525	0.2094	α_{21}	-0.2577	0.1345
$\rho = 0.9$	α_{11}	-0.0870	0.1993	α_{21}	-0.2559	0.1193
$\rho = 0.2$	α_{12}	-0.2258	0.2152	α_{22}	0.0511	0.1448
$\rho = 0.5$	α_{12}	-0.1207	0.2010	α_{22}	0.0623	0.1304
$\rho = 0.9$	α_{12}	-0.0564	0.1795	α_{22}	0.0658	0.1193
$\rho = 0.2$	α_{13}	-0.0038	0.2384	α_{23}	0.0166	0.1261
$\rho = 0.5$	α_{13}	0.0929	0.2308	α_{23}	0.0291	0.1152
$\rho = 0.9$	α_{13}	0.1898	0.2173	α_{23}	0.0373	0.1040
$\rho = 0.2$	α_{14}	-0.0501	0.2197	α_{24}	-0.0532	0.1065
$\rho = 0.5$	α_{14}	0.0054	0.2070	α_{24}	-0.0564	0.1044
$\rho = 0.9$	α_{14}	0.0528	0.2024	α_{24}	-0.0430	0.1043

[a]FRP - First Regime Parameter
[b]SRP - Second Regime Parameter

Table 3: Posterior results using simulated data for TAR model without switches in error term having first autoregressive.

First Regime Parameter	Mean	SD	Second Regime Parameter	Mean	SD
α_{10}	0.4264	0.4271	α_{20}	-0.1065	0.3111
α_{10}	-0.3644	0.3309	α_{20}	-0.9087	0.2461
α_{10}	-0.0438	0.3590	α_{20}	-0.7790	0.2696
α_{11}	-0.0413	0.3505	α_{21}	-0.4954	0.2703
α_{14}	-0.0422	0.3453	α_{24}	-0.1181	0.2896

Table 4: Posterior results using real life data for TAR model without switches in error term having first autoregressive.

First Regime Parameter	Mean	SD	Second Regime Parameter	Mean	SD
α_{10}	0.2896	0.2823	α_{20}	-0.1534	0.3939
α_{10}	-0.2602	0.2974	α_{20}	-0.2452	0.2518
α_{10}	-0.2729	0.2832	α_{20}	0.0802	0.1489
α_{11}	-0.0401	0.2582	α_{21}	0.0323	0.1314
α_{14}	-0.1064	0.2411	α_{24}	-0.0495	0.1063

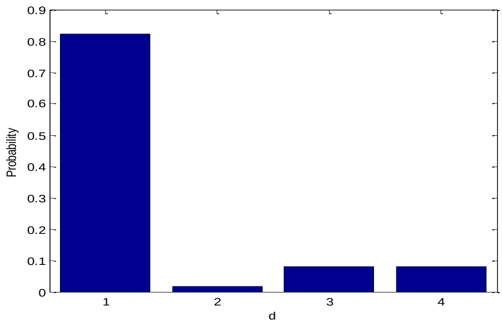

Figure 1: Posterior of delay parameter for simulated data

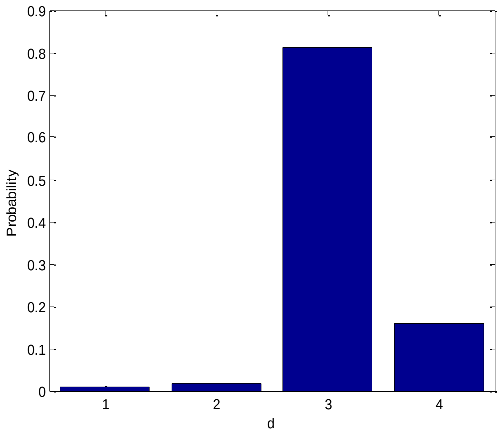

Figure 2: Posterior of delay parameter for real life data

4 Conclusion

Threshold autoregressive model has become attractive to practitioners. Apart from its flexibility in applications, new innovations in regime switching have been developed recently. In this work, we considered a threshold autoregressive when the disturbance terms has a regime switching with the first autoregressive process. The novel Bayesian method was used to evaluate this model. The posterior inference was carried out via Bayesian procedures using both simulated and real life data.

The results obtained from the simulation show that as the ρ increases, the values of P will lead to a better model. This was also supported by the results obtained using real life-data. However, when the threshold autoregressive model with regime switching in disturbance terms with the first autoregressive process was compared with the threshold autoregressive model without regime switching in disturbance term; the results of the former were better than the latter, which implies that the proposed model will make a better forecast. Therefore, practitioners are advised to take advantage of the model.

Conflict of interest

There is no conflict of interest.

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Code availability

The code used can be obtained from the corresponding author.

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