


SAI Method for Solving Job Shop Sequencing Problem under Certain and Uncertain Environment

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ABSTRACT

In this investigation, we use SAI method (Gupta et al. 2016), for solving sequencing problem when processing time of the machine is certain or uncertain in nature. The procedure adopted for solving the sequencing problems is easiest and involves the minimum numbers of iterations to obtain the sequence of jobs. The uncertainty in data is represented by triangular or trapezoidal fuzzy numbers. Yager's ranking function approach is used to convert these fuzzy numbers into a crisp at a prescribed value of α . Stepwise SAI method is then used to obtain optimal job sequence for the problem. Further, the result obtained by SAI method is compared with Johnson's Method. Numerical examples are given to demonstrate the effectiveness of the proposed approach.

Keywords: Sequencing Problem, Johnson's Method, SAI Method, Fuzzy Number, Ranking Method.

1. Introduction

Sequencing problem is considered to be one of the classic and important applications of operations research. The main role of the classical sequencing problem is to find the optimal sequence of the jobs on machines so as to minimize the total amount of time required to complete the process of all the jobs. The simplest pure sequencing problem is one in which there is a single resource or machine, and all processing times are deterministic. The goal of the sequencing problem consists of determining the order or sequence in which the machines will process the jobs so as to optimize some measure of performance (i.e. cost, time or mileage, weight, etc.) to complete the process. The effectiveness of the sequencing problem can be measured in terms of minimized

costs, maximized profits, minimized elapsed time, and meeting due dates etc. In the past, because of its practical and significant use in the production field, many researchers have shown their interest in the sequencing problem. One of the renowned works in the field of sequencing considered till date is by Johnson (1954) who gave the algorithm for production scheduling in which he had minimized the total idle time of machines and the total production times of the jobs. Later Smith and Dudek (1967) developed a general algorithm for the solution of the n -job on m -machine sequencing problem of the flow shop when no passing is allowed. Similarly, Rao *et al.* (2013) and many others gave the technique to minimize the total ideal time of machines or the total production time of the jobs on the two machines production scheduling problems.

A heuristic algorithm for solving general sequencing or flow shop scheduling problem was given by (Nawaz (1983); Ho and Chang (1991); Laha and Chakraborty (2009)) for minimizing elapsed time in no-wait flow-shop scheduling. Baker (2002) considers complete enumeration, integer programming, branch and bound techniques to obtain the optimal sequences, but he does not provide efficient solutions for the large size problems. While Kalczynski and Kamburowski (2006) were dealt with the classical problem of minimizing the makespan in a two-machine flow shop with deterministic job processing times, the optimal job sequence was determined by applying Johnson's rule.

Recently, Ahmad and Khan (2015) gave an algorithm for the constrained flow-shop scheduling problem in which they considered the transportation time, weight of jobs and break down time with m -machines to obtained an optimal or near optimal solution. The pioneering work of Zadeh (1965) concerning the processing of uncertainties by the fuzzy sets has opened a wide range of applications in many diverse fields. Fuzzy rule-based systems have been successfully applied to various applications such as transportation problem, assignment problem, game theory, and soon. Most of the literature deals with a regular measure such as mean flow, time mean lateness, the percentage of jobs tardy, mean tardiness, etc., in deterministic time but the environment in modern society is neither fixed nor probabilistic. So, here we are considering fuzzy environment sequencing problem i.e., the processing time of each job is considered to be indeterministic in nature. The processing time of a job can vary in many ways, may be due to an environmental factor or due to the different workplaces. We find that when a contractor takes the work from a department, he/she calculates total elapsed time for completing the work. But due to many factors like unavailability of labor, weather not favorable, or sometimes abnormal conditions, processing time may vary. In this situation, fuzzy processing time

gives us the realistic idea that helps in making a decision in indeterministic nature. Considering fuzzy processing times yields a more complex scheduling problem since it involves fuzzy arithmetic. Several models with processing times as fuzzy numbers have been used so far.

Dumitru and Luban (1982) extended their job sequencing problem which was formulated as a mathematical programming problem with fuzzy membership functions and fuzzy constraints. Turksen *et al.* (1988) have proposed an approximate reasoning approach to solving a deterministic job shop scheduling problem. McCahon and Lee (1992) have used fuzzy trapezoidal numbers in a flow shop problem and Hejazi *et al.* (2009) introduced an improved version of it. Hong and Chuang (1999) proposed a new triangular fuzzy Johnson algorithm. Chanas and Kasperski (2004) an optimality evaluation of sequences under fuzzy parameters has been investigated. Bagherpouret *al.* (2007) applying fuzzy logic to estimate setup times in sequence-dependent single machine scheduling problems. Mukherjee & Basu (2010) used the ranking method to get the optimal assignment of jobs and Nirmala and Anju (2014) extend this method to get the optimal sequence of fuzzy sequencing problem. Jain and Jain (2011) have presented Fuzzy TOPSIS method in job sequencing problems on machines of unequal efficiencies.

In this investigation, we consider a Job shop sequencing problem under certain and uncertain processing time. SAI method is used to frame a sequence of jobs for processing the n -jobs on m -machines in such a way that the total elapsed time is minimized. The uncertain processing time is converted into a crisp form by Yager's ranking function approach at a prescribed value of α . To check the effectiveness of SAI method, the result obtained is compared with Johnson's Methods.

The remaining contents of this research article are organized in different sections as follows. Section 2, introduces the ranking function approach for converting processing time into crisp form used in the paper. In, Section 3, we use the SAI method for solving the job-shop sequencing problem under fuzzy environment. Numerical examples are provided in section 4. Finally, in last Section 5, we conclude our investigation by summarizing the research work and highlighting the noble features of investigation done.

2. Ranking Function Approach for Converting Processing Time into Crisp form

Generally, in real world job sequencing problems the processing time of a job on the machine is not precisely known but instead, there is vagueness in available data. This vagueness in data can be represented by a pattern of fuzzy numbers Zadeh (1965). Let processing time $\tilde{t}_{ik} = (t_1, t_2, t_3)$ be a triangular fuzzy number then its membership function can be defined as:

$$\mu_{\tilde{t}_{ik}}(t) = \begin{cases} 0, & t \leq t_1 \\ t - t_1 / t_2 - t_1, & t_1 \leq t \leq t_2 \\ t_3 - t / t_3 - t_2, & t_2 \leq t \leq t_3 \\ 0, & t \geq t_3 \end{cases}$$

Now, the interval $\alpha_{\tilde{t}_{ik}}$ is obtained by using α -cut approach

$$t - t_1 / t_2 - t_1 = \alpha, \quad t_3 - t / t_3 - t_2 = \alpha \Rightarrow \begin{cases} t = t_1 + (t_2 - t_1)\alpha \\ t = -(t_3 - t_2)\alpha + t_3 \end{cases}$$

Thus,

$$\alpha \tilde{t}_{ik} = [\tilde{t}_{ik}^L, \tilde{t}_{ik}^U] = [t_1 + (t_2 - t_1)\alpha, -(t_3 - t_2)\alpha + t_3]$$

Such that

$${}^\alpha(\tilde{t}_{ik})^L \leq {}^\alpha(\tilde{t}_{ik}) \leq {}^\alpha(\tilde{t}_{ik})^U$$

Now, Yager (1981) ranking index is used to convert triangular processing time into crisp form:

$$Y(\tilde{t}_{ik}) = \frac{1}{2} \int_0^1 [t_1 + (t_2 - t_1)\alpha, -(t_3 - t_2)\alpha + t_3] d\alpha$$

Let processing time $\tilde{t}_{ik} = (t_1, t_2, t_3, t_4)$ be a trapezoidal fuzzy number then its membership function can be defined as:

$$\mu_{\tilde{t}_{ik}}(t) = \begin{cases} 0, & t \leq t_1 \\ t - t_1 / t_2 - t_1, & t_1 \leq t \leq t_2 \\ 1, & t_2 \leq t \leq t_3 \\ t_4 - t / t_4 - t_3, & t_3 \leq t \leq t_4 \\ 0, & t \geq t_4 \end{cases}$$

Now, the interval $\alpha_{t_{ik}}$ is obtained by using α -cut approach

$$t - t_1 / t_2 - t_1 = \alpha, t_4 - t / t_4 - t_3 = \alpha \Rightarrow \begin{cases} t = t_1 + (t_2 - t_1)\alpha \\ t = -(t_4 - t_3)\alpha + t_4 \end{cases}$$

Thus,

$$\alpha \tilde{t}_{ik} = [\tilde{t}_{ik\alpha}^L, \tilde{t}_{ik\alpha}^U] = [t_1 + (t_2 - t_1)\alpha, -(t_4 - t_3)\alpha + t_4]$$

Such that

$$\alpha (\tilde{t}_{ik})^L \leq \alpha (\tilde{t}_{ik}) \leq \alpha (\tilde{t}_{ik})^U$$

Now, Yager's ranking index is used to convert trapezoidal processing time into crisp form:

$$Y(\tilde{t}_{ik}) = \frac{1}{2} \int_0^1 [t_1 + (t_2 - t_1)\alpha, -(t_4 - t_3)\alpha + t_4] d\alpha$$

3. SAI Methodfor Job-Shop Sequencing Problem under Fuzzy Environment

Here, we considered two cases for solving job shop sequencing problem

Case 3.1: When processing time is precise/certain in nature

The step-wise iterative procedure of SAI method for determining the optimum sequence for n jobs (1,2,..., n) on m machines (1,2,..., m) is as follows:

Step 1: The processing time of n jobs (1,2,..., n) on m machines (1,2,..., m) is given in Table 1.

Machines	Jobs	1	2	3	...	K	...	N
1		t_{11}	t_{12}	t_{13}	\dots	t_{1k}	\dots	t_{1n}
2		t_{21}	t_{22}	t_{23}	\dots	t_{2k}	\dots	t_{2n}
3		t_{31}	t_{32}	t_{33}	\dots	t_{3k}	\dots	t_{3n}
:		$:$	$:$	$:$	$:$	$:$	$:$	$:$
I		t_{i1}	t_{i2}	t_{i3}	\dots	t_{ik}	\dots	t_{in}
:		$:$	$:$	$:$	$:$	$:$	$:$	$:$
M		t_{m1}	t_{m2}	t_{m3}	\dots	t_{mk}	\dots	t_{mn}

Table 1: Processing of n jobs in m machines

Step 2: Examine the jobs and select the least job processing time among all n jobs ($k=1,2,\dots,n$) for each machine and then marked it with $(-)$ sign. Let the minimum processing time occurred at k^{th} job on i^{th} machine.

Mathematically; we can say

$$\min_k \{t_{i1}, t_{i2}, \dots, t_{ik}, \dots, t_{in}\} = t_{ik}$$

Step 3: Similarly, select the least processing time among all m machines ($i=1,2,\dots,m$) for each job and then marked it with $(+)$ sign. Let the minimum processing time is occurred at i^{th} machine for the k^{th} job.

Mathematically; we can say

$$\min_i \{t_{1k}, t_{2k}, \dots, t_{ik}, \dots, t_{mk}\} = t_{ik}$$

Step 4: Examine the rows and columns of Table 1, select the cell with $(+)$ sign has occurred at the cell which corresponds to the i^{th} machine and k^{th} job. The k^{th} job is excluded from the table and is placed first in the optimal job sequence.

Step 5: Step 1 to 4 are repeated until all the jobs are placed in the optimal job sequence.

Consider a situation when a tie has occurred

- i) If $(+)$ occurs at more than one place, then the job with least processing is selected and is placed in the optimal job sequence.
- ii) If $(+)$ occurs at more than one place and the processing time for the allocated jobs is same. Then the job which will process on the lower order positional machine is selected that is by ignoring the other higher order of machines.

Mathematically,

$$\begin{aligned} \min_k \{t_{i1}, t_{i2}, \dots, t_{ik}, \dots, t_{in}\} &= \{t_{i1}, t_{i2}, \dots, t_{ik}\} \quad \forall k = (1, 2, \dots, n) \\ \min_i \{t_{1k}, t_{2k}, \dots, t_{ik}, \dots, t_{mk}\} &= \{t_{(1)k}, t_{(2)k}, \dots, t_{(i)k}\} \quad \forall i = (1, 2, \dots, n) \end{aligned}$$

Step 7: Lastly, we calculate the ideal time and total elapsed time of machines.

Case 3.2: When processing time is imprecise/uncertain in nature

In most of the real life situations, the processing time of a job on a machine is usually not precisely or exactly known but there can be a rough idea of the time pattern of the processing time of job-based on the previously done works on the machines. In such situation where the information cannot be represented by one exact value but there is vagueness in data. In such situation, the concept of fuzzy numbers of fuzzy set theory can be applied. So, here we consider job shop scheduling problem with a fuzzy processing time of jobs on machines and solve it by SAI method.

The step-wise iterative procedure is given below:

Step 1: Let the processing time of an i^{th} job on k^{th} machine be a trapezoidal fuzzy number represented as $\tilde{t}_{ik} = (t_1, t_2, t_3, t_4)$ where $i = 1, 2, \dots, m$ and $k = 1, 2, \dots, n$. Fuzzy Processing time for n jobs on m machines can be summarized as shown in the table below:

Machines	Jobs	1	2	3	...	K	...	N
1		\tilde{t}_{11}	\tilde{t}_{12}	\tilde{t}_{13}	...	\tilde{t}_{1k}	...	\tilde{t}_{1n}
2		\tilde{t}_{21}	\tilde{t}_{22}	\tilde{t}_{23}	...	\tilde{t}_{2k}	...	\tilde{t}_{2n}
3		\tilde{t}_{31}	\tilde{t}_{32}	\tilde{t}_{33}	...	\tilde{t}_{3k}	...	\tilde{t}_{3n}
:		:	:	:	:	:	:	:
I		\tilde{t}_{i1}	\tilde{t}_{i2}	\tilde{t}_{i3}	...	\tilde{t}_{ik}	...	\tilde{t}_{in}
:		:	:	:	:	:	:	:
M		\tilde{t}_{m1}	\tilde{t}_{m2}	\tilde{t}_{m3}	...	\tilde{t}_{mk}	...	\tilde{t}_{mn}

Table 2: Processing of n jobs on m machines

Step 2: Convert the fuzzy processing time for each job into crisp form by using Yager's ranking index. Let the fuzzy processing time of \tilde{t}_{11} be defined as $\tilde{t}_{11} = (t_1, t_2, t_3, t_4)$, where $t_1 \leq t_2 \leq t_3 \leq t_4$

$$Y(\tilde{t}_{11}) = \int_0^1 0.5(t_\alpha^L + t_\alpha^U) d\alpha = \int_0^1 0.5[(t_2 - t_1)\alpha + t_1, -(t_4 - t_3)\alpha + t_4] d\alpha$$

Similarly for

$$Y(\tilde{t}_{ik}) = \int_0^1 0.5(t_\alpha^L + t_\alpha^U) d\alpha = \int_0^1 0.5[(t_2 - t_1)\alpha + t_1, -(t_4 - t_3)\alpha + t_4] d\alpha \text{ and soon.}$$

Step 3: Now, construct the table 2 by replacing fuzzy processing time of each job on each machine by its calculated crisp value as shown below:

↓ Machines	Jobs →	1	2	3	...	K	...	N
1		$Y(\tilde{t}_{11})$	$Y(\tilde{t}_{12})$	$Y(\tilde{t}_{13})$...	$Y(\tilde{t}_{1k})$...	$Y(\tilde{t}_{1n})$
2		$Y(\tilde{t}_{21})$	$Y(\tilde{t}_{22})$	$Y(\tilde{t}_{23})$...	$Y(\tilde{t}_{2k})$...	$Y(\tilde{t}_{2n})$
3		$Y(\tilde{t}_{31})$	$Y(\tilde{t}_{32})$	$Y(\tilde{t}_{33})$...	$Y(\tilde{t}_{3k})$...	$Y(\tilde{t}_{3n})$
:		:	:	:	:	:	:	:
I		$Y(\tilde{t}_{i1})$	$Y(\tilde{t}_{i2})$	$Y(\tilde{t}_{i3})$...	$Y(\tilde{t}_{ik})$...	$Y(\tilde{t}_{in})$
:		:	:	:	:	:	:	:
M		$Y(\tilde{t}_{m1})$	$Y(\tilde{t}_{m2})$	$Y(\tilde{t}_{m3})$...	$Y(\tilde{t}_{mk})$...	$Y(\tilde{t}_{mn})$

Table 3: Crisp Processing time of n jobs on m machines

After obtaining the crisp processing time, SAI method can be applied to Table 3 to determine the optimal sequence for the given problem. The same procedure will be followed for the triangular fuzzy number.

4. Numerical Example

Case 4.1: When processing time is precise/certain in nature

Example 1: N Jobs and 2 Machines Problem

There are 5 jobs, each of which must go through the two machines A and B in the order AB. Processing time is given in Table 4.

↓ Machines	Jobs →	1	2	3	4	5
A		8	6	4	9	3
B		12	9	13	6	2

Table 4: Processing of 5 jobs in 2 machines

By applying SAI method on Table 4, we obtain flow of jobs through machines A and B in the sequence

5	3	2	4	1
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Flow of jobs through machines A and B using the optimal sequence

$$5 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$$

The minimum total elapsed time is calculated for the obtained sequence in table 5 as follows

Sequence	Machine A		Machine B	
	Time In	Time Out	Time In	Time Out
5	0	2	2	3
3	2	5	5	13
2	5	10	13	21
4	10	18	21	26
1	18	25	26	37

Table 5: Computation of total elapsed time for the job sequence

Comparison between SAI and Johnson's method

By using SAI Method, we have

Total elapsed time= 47 hours

Idle Time for Machine A= 17 hours

Idle Time for Machine B= 05 hours

By using Johnson's Algorithm, we have

Total elapsed time=47hours

Idle Time for Machine A= 22 hours

Idle Time for Machine B= 04 hours

Example 2: N Jobs and 3 Machines Problem

There are 5 jobs, each of which must go through the three machines A, B and C in the order ABC. Processing time is given below:

↓Machines	Jobs →	1	2	3	4	5
A		4	9	8	6	3
B		4	5	3	2	6
C		6	9	11	8	7

Table 6: Processing of 5 jobs in 3 machines

By applying SAI method on table 6, we obtain flow of jobs through machines A, B and C in the sequence

$$4 \rightarrow 5 \rightarrow 3 \rightarrow 1 \rightarrow 2$$

Comparison between SAI and Johnson's method

By using SAI Method, we have

Total elapsed time= 46 hours

Idle Time for Machine A= 16 hours

Idle Time for Machine B= 26 hours

Idle Time for Machine C= 11 hours

By using Johnson's Algorithm, we have

Total elapsed time= 49 hours

Idle Time for Machine A= 19 hours

Idle Time for Machine B= 29 hours

Idle Time for Machine C= 08 hours

Example 3: N Jobs and M Machines Problem

There are 4 jobs, each of which must go through the four machines A, B, C and D in the order ABCD. Processing time is given below

Jobs ↓ Machines →	A	B	C	D
1	21	11	10	21
2	18	8	16	18
3	22	9	11	22
4	26	6	10	26

Table 7: Processing of 4 jobs in 4 machines

By applying SAI method on table 7, we obtain flow of jobs through machines A, B, C and D in the sequence

$$4 \rightarrow 2 \rightarrow 3 \rightarrow 1$$

Comparison between SAI and Johnson's method

By using SAI Method, we have

Total elapsed time= 129 hours

Idle Time for Machine A= 42 hours

Idle Time for Machine B= 95 hours

Idle Time for Machine C= 82 hours

Idle Time for Machine D= 42 hours

By using Johnson's Algorithm, we have

Total elapsed time= 129 hours

Idle Time for Machine A= 68 hours

Idle Time for Machine B= 95 hours

Idle Time for Machine C= 82 hours

Idle Time for Machine D= 42 hour

Case 4.2: When processing time is imprecise/uncertain in nature**4.2.1 When processing time is represented by trapezoidal fuzzy number****Example 4: N Jobs and 2 Machines Problem:**

There are 5 jobs, each of which must go through the two machines A and B in the order AB. Fuzzy processing time (hours) are given in Table 8.

Jobs → Machines ↓	1	2	3	4	5
A	(6,7,8,9)	(4,5,6,9)	(2,4,5,7)	(6,8,12,14)	(0,2,4,6)
B	(10,11,13,15)	(6,7,11,15)	(10,12,14,18)	(0,6,7,9)	(0,1,2,3)

Table 8: Fuzzy processing time of 5 jobs on 2 machines

Now, we transform the fuzzy processing time into crisp value by using Yager's Ranking method. The membership function for the trapezoidal fuzzy processing

time $\tilde{t}_{11} = (6, 7, 8, 9)$ is given as:

$$\mu(t) = \begin{cases} 0, & t \leq 6 \\ t - 6/7 - 6, & 6 \leq t \leq 7 \\ 1, & 7 \leq t \leq 8 \\ 9 - t/9 - 8, & 8 \leq t \leq 9 \\ 0, & t \geq 9 \end{cases}$$

The α -cut of the fuzzy processing time $(6, 7, 8, 9)$ is
 $(t_\alpha^L, t_\alpha^U) = ((7-6)\alpha + 6, -(9-8)\alpha + 9) = (\alpha + 6, -\alpha + 9)$

Therefore,

$$\begin{aligned} Y(\tilde{t}_{11}) &= Y(6, 7, 8, 9) = \int_0^1 0.5(t_\alpha^L + t_\alpha^U) d\alpha = \int_0^1 0.5(\alpha + 6 - \alpha + 9) d\alpha \\ &= \int_0^1 0.5 \times 15 d\alpha = 7.50 \end{aligned}$$

Similarly, the other fuzzy processing times using the Yager's indices is calculated:

$$\begin{aligned} Y(\tilde{t}_{12}) &= 6.0, \quad Y(\tilde{t}_{13}) = 4.5, \quad Y(\tilde{t}_{14}) = 10.0, \quad Y(\tilde{t}_{15}) = 3.0, \quad Y(\tilde{t}_{21}) = 12.25, \\ Y(\tilde{t}_{22}) &= 9.75, \quad Y(\tilde{t}_{23}) = 13.50, \quad Y(\tilde{t}_{24}) = 6.5, \quad Y(\tilde{t}_{25}) = 1.5. \end{aligned}$$

Change the fuzzy processing times by crisp value; we have the following crisp sequencing problem

Machines	Jobs	1	2	3	4	5
A		7.50	6.0	4.5	10.0	3.0
B		12.25	9.75	13.50	6.50	1.5

Table 9: Crisp processing time of 5 jobs on 2 machines

By applying the SAI method on crisp sequencing problem, we get the following optimal sequence of jobs through machines A and B

$$5 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$$

Comparison between SAI and Johnson's Method

By using SAI Method, we have

Total elapsed time= 49.50 hours

Idle Time for Machine A= 18.5 hours

Idle Time for Machine B= 6.0 hours

By using Johnson's Algorithm, we have

Total elapsed time= 51.50 hours

Idle Time for Machine A= 20.5 hours

Idle Time for Machine B= 4.5 hours

Example 5: N Jobs and 3 Machines Problem:

There are 5 jobs, each of which must go through the three machines A, B and C in the order ABC. Processing time (hours) is given below.

Jobs → ↓ Machines	1	2	3	4	5
A	(0,3,4,5)	(7,8,10,12)	(5,7,8,10)	(2,5,6,10)	(1,2,3,4)
B	(2,3,5,7)	(3,4,7,10)	(0,2,4,8)	(0,1,3,6)	(3,5,6,9)
C	(3,5,6,8)	(5,8,9,11)	(6,10,11,13)	(4,7,11,15)	(6,9,10,13)

Table 10: Fuzzy processing of 5 jobs on 3 machines

Using Yager's ranking index fuzzy processing time transform into crisp value

$$Y(\tilde{t}_{11}) = 0.3, \quad Y(\tilde{t}_{12}) = 9.25, \quad Y(\tilde{t}_{13}) = 7.5, \quad Y(\tilde{t}_{14}) = 5.5, \quad Y(\tilde{t}_{15}) = 2.5, \\ Y(\tilde{t}_{21}) = 4.25, \quad Y(\tilde{t}_{22}) = 6.0, \quad Y(\tilde{t}_{23}) = 3.5, \quad Y(\tilde{t}_{24}) = 2.5, \quad Y(\tilde{t}_{25}) = 5.75, \\ Y(\tilde{t}_{31}) = 5.5, \quad Y(\tilde{t}_{32}) = 8.0, \quad Y(\tilde{t}_{33}) = 10.0, \quad Y(\tilde{t}_{34}) = 9.25, \quad Y(\tilde{t}_{35}) = 9.5.$$

Change the fuzzy processing times by crisp value; we have the following crisp sequencing problem:

↓ Machines	Jobs →	1	2	3	4	5
A		3.0	9.25	7.5	5.5	2.5
B		4.25	6.0	3.5	2.5	5.75
C		5.5	8.0	10.0	9.25	9.5

Table 11: Crisp processing of 5 jobs on 3 machines

By applying the SAI method on crisp sequencing problem, we get the following optimal sequence of jobs through machines A, B and C

$$4 \rightarrow 5 \rightarrow 3 \rightarrow 1 \rightarrow 2$$

Comparison between SAI and Johnson's Method

By using SAI Method, we have

Total elapsed time= 50.50 hours
Idle Time for Machine A= 22.75 hours
Idle Time for Machine B= 28 hours
Idle Time for Machine C= 8.25 hours

By using Johnson's Algorithm, we have

Total elapsed time= 50.50 hours
Idle Time for Machine A= 21.75 hours
Idle Time for Machine B= 27.5 hours
Idle Time for Machine C= 7.25 hours

Example 6: N Jobs and M Machines Problem

There are 4 jobs, each of which must go through the four machines A, B, C and D in the order ABCD.

Processing time is given below

Jobs → ↓ Machines	1	2	3	4
A	(18,20,21,23)	(16,17,19,20)	(19,21,24,26)	(23,15,27,29)
B	(7,10,11,13)	(5,7,9,10)	(4,8,10,12)	(3,5,7,9)
C	(7,9,11,13)	(12,15,16,19)	(9,10,12,13)	(7,9,11,13)
D	(18,20,21,23)	(16,17,19,20)	(19,21,24,26)	(23,25,27,29)

Table 12: Fuzzy processing of 5 jobs on 4 machines

Using Yager's ranking index fuzzy processing time transform into crisp value

$$Y(\tilde{t}_{11}) = 20.5, \quad Y(\tilde{t}_{12}) = 18.0, \quad Y(\tilde{t}_{13}) = 22.5, \quad Y(\tilde{t}_{14}) = 26.0, \quad Y(\tilde{t}_{21}) = 10.25, \\ Y(\tilde{t}_{22}) = 7.75, \quad Y(\tilde{t}_{23}) = 8.0, \quad Y(\tilde{t}_{24}) = 6.0, \quad Y(\tilde{t}_{31}) = 9.75, \quad Y(\tilde{t}_{32}) = 15.5, \\ Y(\tilde{t}_{33}) = 10.25, \quad Y(\tilde{t}_{34}) = 9.25, \quad Y(\tilde{t}_{41}) = 20.5, \quad Y(\tilde{t}_{42}) = 18.0, \quad Y(\tilde{t}_{43}) = 22.5, \\ Y(\tilde{t}_{44}) = 26.0.$$

Change the fuzzy processing times by crisp value; we have the following crisp sequencing problem

Machines ↓	Jobs →	1	2	3	4
A		20.5	18.0	22.5	26.0
B		10.25	7.75	8.0	6.0
C		9.75	15.5	10.25	9.75
D		20.5	18.0	22.5	26.0

Table 13: Crisp processing of 5 jobs on 4 machines

By applying the SAI method on fuzzy processing times, we get the following optimal sequence of jobs through machines A, B, C and D

$$4 \rightarrow 2 \rightarrow 3 \rightarrow 1$$

Comparison between SAI and Johnson's Method

By using SAI Method, we have

Total elapsed time= 128.75 hours
Idle Time for Machine A= 41.75 hours
Idle Time for Machine B= 96.75 hours
Idle Time for Machine C= 84 hours
Idle Time for Machine D= 41.75 hours

By using Johnson's Algorithm, we have

Total elapsed time= 128.75 hours
Idle Time for Machine A= 41.75 hours
Idle Time for Machine B= 96.75 hours
Idle Time for Machine C= 84 hours
Idle Time for Machine D= 41.75 hour

4.2.2 When Processing Time is represented by Triangular fuzzy number

Example 7: N Jobs and 2 Machines Problem:

There are 5 jobs, each of which must go through the two machines A and B in the order AB.

Fuzzy processing time (hours) are given below.

↓ Machines	Jobs →	1	2	3	4	5
A		(6,7,8)	(4,5,6)	(2,4,5)	(6,8,12)	(0,2,4)
B		(10,11,13)	(6,7,11)	(10,12,14)	(0,6,7)	(0,1,2)

Table 14: Fuzzy processing time of 5 jobs on 2 machines

Now, we transform the fuzzy processing time into crisp value by using Yager's Ranking method. The membership function for the trapezoidal fuzzy processing time $\tilde{t}_{11} = (6, 7, 8)$ is given as:

$$\mu(t) = \begin{cases} t-6/7-6, & 6 \leq t \leq 7 \\ 8-t/8-7, & 7 \leq t \leq 8 \\ 0, & t \geq 8 \end{cases}$$

The α -cut of the fuzzy processing time $(6, 7, 8, 9)$ is

$$(t_{\alpha}^L, t_{\alpha}^U) = ((7-6)\alpha + 6, -(8-7)\alpha + 8) = (\alpha + 6, -\alpha + 8)$$

Therefore,

$$\begin{aligned} Y(\tilde{t}_{11}) &= Y(6, 7, 8, 9) = \int_0^1 0.5(t_{\alpha}^L + t_{\alpha}^U) d\alpha = \int_0^1 0.5(\alpha + 6 - \alpha + 8) d\alpha \\ &= \int_0^1 0.5 \times 14 d\alpha = 7. \end{aligned}$$

Similarly, the other fuzzy processing times using the Yager's indices is calculated:

$$Y(\tilde{t}_{12}) = 5.0, \quad Y(\tilde{t}_{13}) = 3.75, \quad Y(\tilde{t}_{14}) = 8.50, \quad Y(\tilde{t}_{15}) = 2.0, \quad Y(\tilde{t}_{21}) = 11.25, \\ Y(\tilde{t}_{22}) = 7.75, Y(\tilde{t}_{23}) = 12.00, Y(\tilde{t}_{24}) = 4.75, Y(\tilde{t}_{25}) = 1.0.$$

Change the fuzzy processing times by crisp value; we have the following crisp sequencing problem

↓ Machines	Jobs →	1	2	3	4	5
A		7.0	5.0	3.75	8.50	2.0
B		11.25	7.75	12.00	4.75	1.0

Table 15: Crisp processing time of 5 jobs on 2 machines

By applying the SAI method on crisp sequencing problem, we get the following optimal sequence of jobs through machines A and B

$$5 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$$

Comparison between SAI and Johnson's Method

By using SAI Method, we have

Total elapsed time= 40.50 hours

Idle Time for Machine A= 14.25 hours

Idle Time for Machine B= 4.75 hours

By using Johnson's Algorithm, we have

Total elapsed time= 41.50 hours

Idle Time for Machine A= 15.25 hours

Idle Time for Machine B= 4.75 hours

Example 8: N Jobs and 3 Machines Problem:

There are 5 jobs, each of which must go through the three machines A, B and C in the order ABC. Processing time (hours) is given below.

Machines	Jobs	1	2	3	4	5
A		(0,3,4)	(7,8,10)	(5,7,8)	(2,5,6)	(1,2,3)
B		(2,3,5)	(3,4,7)	(0,2,4)	(0,1,3)	(3,5,6)
C		(3,5,6)	(5,8,9)	(6,10,11)	(4,7,11)	(6,9,10)

Table 16: Fuzzy processing of 5 jobs on 3 machines

Using Yager's ranking index fuzzy processing time transform into crisp value

$$Y(\tilde{t}_{11}) = 2.50, Y(\tilde{t}_{12}) = 8.25, Y(\tilde{t}_{13}) = 6.75, Y(\tilde{t}_{14}) = 4.50, Y(\tilde{t}_{15}) = 2.0,$$

$$Y(\tilde{t}_{21}) = 3.25, Y(\tilde{t}_{22}) = 4.50, Y(\tilde{t}_{23}) = 2.0, Y(\tilde{t}_{24}) = 1.25, Y(\tilde{t}_{25}) = 4.75,$$

$$Y(\tilde{t}_{31}) = 4.75, Y(\tilde{t}_{32}) = 7.50, Y(\tilde{t}_{33}) = 9.25, Y(\tilde{t}_{34}) = 7.25, Y(\tilde{t}_{35}) = 8.50.$$

Change the fuzzy processing times by crisp value; we have the following crisp sequencing problem

Machines	Jobs	1	2	3	4	5
A		2.50	8.25	6.75	4.50	2.0
B		3.25	4.50	2.0	1.25	4.75
C		4.75	7.50	9.25	7.25	8.50

Table 17: Crisp processing of 5 jobs on 3 machines

By applying the SAI method on crisp sequencing problem, we get the following optimal sequence of jobs through machines A, B and C

$$4 \rightarrow 5 \rightarrow 3 \rightarrow 1 \rightarrow 2$$

Comparison between SAI and Johnson's Method

By using SAI Method, we have

Total elapsed time= 43 hours

Idle Time for Machine A= 19 hours

By using Johnson's Algorithm, we have

Total elapsed time= 43 hours

Idle Time for Machine A= 19 hours

Idle Time for Machine B= 27 hours
Idle Time for Machine C= 5.75 hours

Idle Time for Machine B= 27.25 hours
Idle Time for Machine C= 5.75 hours

Example 9: N Jobs and M Machines Problem

There are 4 jobs, each of which must go through the four machines A, B, C and D in the order ABCD. Processing time is given below

↓Machines	Jobs→	1	2	3	4
A		(18,20,21)	(16,17,19)	(19,21,24)	(23,15,27)
B		(7,10,11)	(5,7,9,10)	(4,8,10)	(3,5,7)
C		(7,9,11)	(12,15,16)	(9,10,12)	(7,9,11)
D		(18,20,21)	(16,17,19)	(19,21,24)	(23,25,27)

Table 18: Fuzzy processing of 5 jobs on 4 machines

Using Yager's ranking index fuzzy processing times transform into crisp value
 $Y(\tilde{t}_{11}) = 19.75$, $Y(\tilde{t}_{12}) = 17.25$, $Y(\tilde{t}_{13}) = 21.25$, $Y(\tilde{t}_{14}) = 25.0$, $Y(\tilde{t}_{21}) = 9.50$,
 $Y(\tilde{t}_{22}) = 7.0$, $Y(\tilde{t}_{23}) = 7.50$, $Y(\tilde{t}_{24}) = 5.0$, $Y(\tilde{t}_{31}) = 9.0$, $Y(\tilde{t}_{32}) = 14.50$,
 $Y(\tilde{t}_{33}) = 10.25$, $Y(\tilde{t}_{34}) = 9.0$, $Y(\tilde{t}_{41}) = 19.75$, $Y(\tilde{t}_{42}) = 17.25$, $Y(\tilde{t}_{43}) = 21.25$,
 $Y(\tilde{t}_{44}) = 25.0$.

Change the fuzzy processing times by crisp value; we have the following crisp sequencing problem

↓Machines	Jobs→	1	2	3	4
A		19.75	17.25	21.25	25.0
B		9.50	7.0	7.50	5.0
C		9.0	14.50	10.25	9.0
D		19.75	17.25	21.25	25.0

Table 19: Crisp processing of 5 jobs on 4 machines

By applying the SAI method on fuzzy processing times, we get the following optimal sequence of jobs through machines A, B, C and D

$$4 \rightarrow 2 \rightarrow 3 \rightarrow 1$$

Comparison between SAI and Johnson's Method

By using SAI Method, we have

Total elapsed time= 122.25 hours
Idle Time for Machine A= 38.50 hours
Idle Time for Machine B= 93.25 hours
Idle Time for Machine C= 79.50 hours
Idle Time for Machine D= 39 hours

By using Johnson's Algorithm, we have

Total elapsed time= 122.25 hours
Idle Time for Machine A= 38.50 hours
Idle Time for Machine B= 93.25 hours
Idle Time for Machine C= 79.50 hours
Idle Time for Machine D= 39 hour

Special Case: N Jobs and M Machines Problem

There are 5 jobs, each of which must go through the four machines A, B, C and D in the order ABCD. Processing time is given below

Machines	Jobs	1	2	3	4	5
A		8	12	3	15	19
B		16	19	14	5	12
C		15	19	12	28	33
D		22	7	17	15	17

Table 20: Fuzzy processing of 5 jobs on 4 machines

By applying Johnson's method on table 19, we first convert the problem into two machine problem by adopting the following steps

Step 1: $\min_i (A_i, D_i) = (3, 7)$ and $\max_i (B_i, C_i) = (19, 33)$

Step 2: The inequality

$$\min_i A_i = 3 \geq \max_i (B_i, C_i) \text{ not satisfied}$$

$$\min_i D_i = 7 \geq \max_i (B_i, C_i) \text{ not satisfied}$$

Since none of the inequalities in step 2 are satisfied, Johnson's method cannot be applied to this problem but by applying SAI method on Table 19, we obtain flow of jobs through machines A, B, C and D in the sequence

$$3 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 5$$

Total elapsed time= 141 hours

Idle Time for Machine A= 84 hours

Idle Time for Machine B= 75 hours

Idle Time for Machine C= 34 hours

Idle Time for Machine D= 63 hours

5. Conclusion

In this present study, we have considered a job shop sequencing problem with certain and uncertain processing time. The fuzziness in data is handled by Yager's ranking function approach at a prescribed value of α and then SAI method has been used for providing optimal job sequence for n jobs on m machines directly in a minimum number of iterations. Our results show that the SAI method can be easily applied to the small job as well as complex job scheduling problems and also reduces the complexities in solving the job

sequencing problem and also provides the optimal solution easily in less number of iterations in a very short period of time. Thus it can be concluded that the SAI method is powerful, time-saving, and easy to compute.

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